Programming the Kennedy Receiver for Capacity Maximization versus Minimizing One-shot Error Probability

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Abstract: We find the capacity attained by the Kennedy receiver for coherent-state BPSK when the symbol prior p and pre-detection displacement β are optimized. The optimal β is different than what minimizes error probability for single-shot BPSK state discrimination.

1. Optical communication capacity with coherent-state BPSK signaling

Let us consider coherent-state binary phase shift keying (BPSK) symbols $|\alpha\rangle, |-\alpha\rangle$, with priors $(p, 1-p)$ and mean photon number per mode, $|\alpha|^2 = N$, at the receiver. Sam Dolinar found the design of an optical receiver that uses a coherent-state local oscillator (LO), linear optics, switches, and quantum-noise-limited photon detection, which achieves the minimum average probability of error of discriminating the two states, $P_{e,\min}(p) = \frac{1}{2}[1 - \sqrt{1-4p(1-p)e^{-4N}}]$ [1]. For communicating with a BPSK alphabet, if a receiver must detect one BPSK symbol at a time, setting $p = 1/2$, and using the Dolinar receiver achieves the maximum capacity, $C_1(N) = 1 - h_2(P_{e,\min}(1/2))$ bits per BPSK symbol, which is the capacity of a binary symmetric channel of crossover probability $P_{e,\min}(1/2)$. If the receiver is allowed to make joint (quantum, collective) measurements over long codeword blocks, not describable by any symbol by symbol measurement and post-processing, the maximum attainable capacity, $C_\infty(N) = h_2([1-e^{-2N}]/2)$ bits/symbol, is given by the Holevo limit [2]. In this article, we will restrict our attention to a specific (symbol-by-symbol) receiver design [3] which is an extension of a receiver proposed by Robert Kennedy [4]. Hence our attained capacity will stay strictly below C_1. This generalized Kennedy (GK) receiver first applies a coherent displacement to shift the received BPSK symbols to $|\beta\rangle, |2\alpha+\beta\rangle$, and then detects the shifted states with a shot-noise limited photon detector. If the detector clicks, the receiver guesses the "$|\alpha\rangle$" hypothesis, and if it generates no clicks, it guesses the "$|-\alpha\rangle$" hypothesis. The pre-detection optical displacement can be realized by interfering the BPSK symbol on a beamsplitter of transmissivity $\kappa \approx 1$, with a strong coherent state LO $|\beta/\sqrt{1-\kappa}\rangle$. In Fig. 1, left, we show the 2-by-2 transition probability matrix $p_{Y|X}(y|x)$ induced by the GK receiver. We calculate the capacity attained by the Kennedy receiver (by optimizing over the priors p and the displacement β), i.e., $C_{\text{Kennedy}}(N) = \max_{p,\beta} I(X;Y)$. The plots in Fig. 1 (right) show that the Capacity attained by the Kennedy receiver with optimized prior and displacement is higher than that achieved by Kennedy's original (exact displacement) receiver ($\beta = 0$), with p optimized. The improvement is more pronounced when N is small, the same regime where $C_\infty(N)/C_1(N) \to \infty$, of interest, e.g., for deep-space communications.

2. Discussion of results: optimizing capacity versus minimizing the one-shot probability of error

We now discuss our results in the context of important conceptual points regarding designing a receiver for maximizing (communication) rate, versus minimizing one-shot probability of error (e.g., in a target-detection radar).
(a) Error probabilities for one-shot state discrimination. Figure 2a shows average error probability for one-shot BPSK state discrimination, assuming equal priors, $p = 1/2$. The quantum (Helstrom) limit is attained by the Dolinar receiver. The Kennedy receiver's performance is shown, with optimized β. In Fig. 2, we see that the optimal β is quite different than what minimizes error probability for single-shot BPSK state discrimination. Next, we note that Homodyne receiver's probability of error performance is strictly worse than the Kennedy receiver with optimized β, whereas there is a crossover between Homodyne and Kennedy receivers' performance when $\beta = 0$ is set. For channel capacity (see Fig. 1), both for $\beta = 0$ and β optimized for capacity, homodyne receiver does inferior to the Kennedy receiver. This shows that an optical receiver may need to be "programmed" differently based on the information-processing task at hand and that optimizing a receiver to minimize symbol error probability may not result in a capacity-maximizing setting for that same receiver, and vice versa.
(b) Finite blocklength communications rate. Bondurant generalized Kennedy's receiver to the M-ary phase-shift keying (PSK), later generalized to any M-ary constellation. Experiments have been conducted as well [5, 6]. This

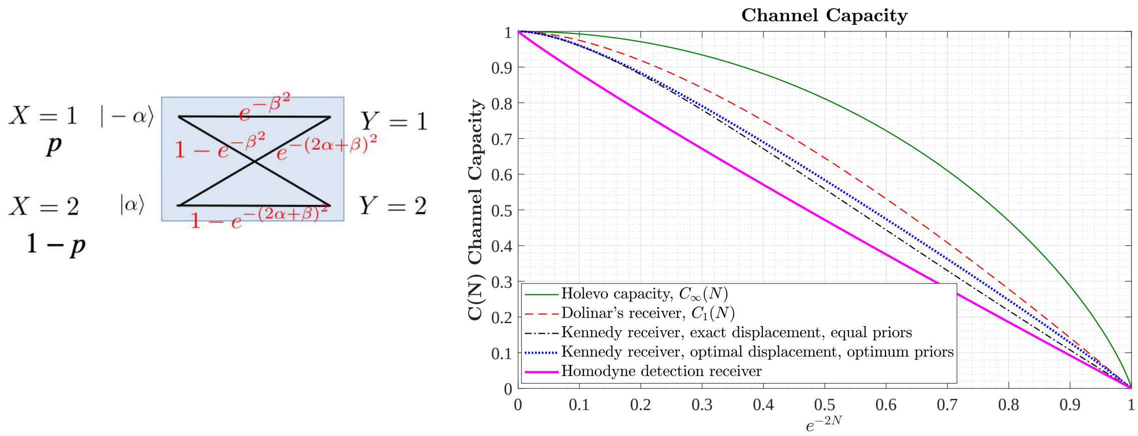
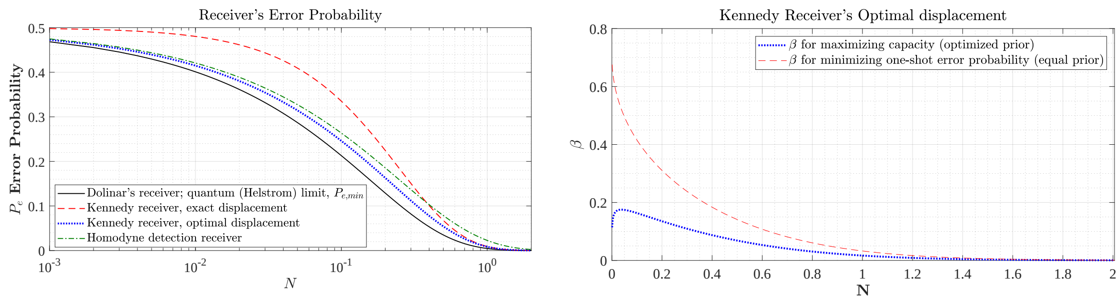


Fig. 1: [Left] Transition Probability Matrix for BPSK signaling and the Generalized Kennedy (GK) receiver. [Right] BPSK communication capacity attained by various receiver choices, in units of bits transmitted per BPSK symbol (bits per mode).

generalized receiver, known as sequential waveform nulling (SWN), does not meet the quantum limit of minimum error state discrimination. But it achieves the optimal error exponent, which is a factor of four higher than that achievable with ideal heterodyne detection [7]. The symbol demodulation error probability $P_{e,\text{Het}} \sim e^{-\xi N}$, $P_{e,\text{SWN}} \sim e^{-4\xi N}$, and $P_{e,\text{opt}} \sim e^{-4\xi N}$, as $N \to \infty$. Here N is the average photon-number of the states in the constellation. In the high N regime, there is not much capacity improvement by using SWN (over ideal heterodyne). But SWN's superior error exponent for symbol demodulation translates to a higher reliability function, i.e., a higher number of bits being transmissible over n channel uses for a given target codeword error probability [8]. This could translate to a higher data volume being transmissible over a dynamic optical link that is only available for a short time duration. In ongoing work, we are optimizing displacement and nulling-order for M-ary constellations to maximize the finite-length rate, by optimizing the channel dispersion attained by the SWN receiver [9].



(a)

(b)

Fig. 2: Figure 2a shows average error probability for one-shot BPSK state discrimination, assuming equal priors. The quantum limit is given by the Helstrom limit, attained by Dolinar's receiver, which also attains capacity C_1, the optimal within all receivers that detect each BPSK symbol one by one. Figure 2b compares the β, optimal displacement for the Kennedy receiver, for: (a) optimizing capacity (red dashed), and (b) minimizing error probability assuming equal priors (blue dotted).

3. Acknowledgment

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